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THE MODEL PROBLEM OF CONTROLLING THE LATERAL MOTION OF AN AIRCRAFT DURING LANDING*

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The problem of controlling the lateral motion of an aircraft during its final landing stages under windy conditions is studied in the linear approximation. The problem is formalized in the form of an antagonistic two-person positional differential game with a fixed time of termination and convex payoff function. Results of a numerical solution are described. The paper is related to /1—5/.

1. The lateral motion of a medium-size transport plane in its final approach to landing can be described in the linear approximation by the following differential vector equation /3, 6/:

$$x^{\cdot} = Ax + Bu + Cv, \quad x \in R^7 \tag{1.1}$$

$$A = \left\| \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0{,}0762 & -5{,}34 & 0 & 9{,}81 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -0{,}0056 & -0{,}392 & -0{,}0889 & -0{,}0378 & -0{,}17 & 0{,}0378 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -0{,}0129 & -0{,}9016 & -0{,}2045 & -0{,}0869 & -0{,}89 & 0{,}0869 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k \end{matrix} \right\|$$

$$B = (0;\,0;\,0;\,0;\,0;\,0;\,k)',\; C = (0;\,0{,}0762;\;0;\;0{,}0056;\;0;\,0{,}0129;\,0)'$$

The first component x_1 of the phase vector describes the lateral deviation of the centre of mass of the plane from the axis of the runway, x_2 is the rate of lateral deviation, x_3 is the yaw angle (measured from the runway axis), x_4 is the rate of change of the yaw angle, x_5 is the roll angle and x_6, x_7 are the auxiliary variables. The control parameter u is treated as a given roll angle /7/ and the parameter v as the lateral component of the wind velocity. The coefficient k in the matrix A and column B characterises the inertia of the process of tracking the prescribed roll and is assumed to be unchanged during the motion. The lateral deviation is measured in meters, the angles in radiants, and the time in seconds.

We shall study the behaviour of system (1.1) in the time interval $[0, \vartheta]$ where ϑ is the instant of passing the runway and is to be assumed constant. The assumption is justified, if we suppose that the lengthwise motion of the plane is independent of the lateral motion and takes place according to a prescribed program /7, 8/. Henceforth, we shall put $\vartheta = 15$ sec.

Safety considerations limit the prescribed roll angle in modulo, and the amount of restriction depends on the altitude /7/. Assuming that the altitude is related uniquely to t, we shall write /3/

$$|u| \leqslant \mu(t) = 0{,}2613 - 0{,}0116t \quad \text{radians} \tag{1.2}$$

We shall assume that the restriction imposed on the magnitude of the lateral component of the wind velocity is independent of t. Suppose

$$|v| \leqslant \nu = 10 \quad \text{m/sec.} \tag{1.3}$$

We shall say that the parameter u belongs to the first player, and v to the second. We define in the plane of the phase variable x_1, x_2 the set

$$M = \left\{(x_1, x_2): \frac{x_1^2}{216} - \frac{2x_1}{9} - \frac{3}{2} \leqslant x_2 \leqslant -\frac{x_1^2}{216} - \frac{2x_1}{9} + \frac{3}{2}\right\}$$

and introduce the payoff function

$$\varphi(x_1, x_2) = \min\{c \geqslant 0\colon (x_1, x_2) \in cM\} \tag{1.4}$$

We will assume that the purpose of the first player is to minimize the values $\varphi(x_1(\vartheta), x_2(\vartheta))$, and the second player has the opposite interest. The set M is symmetrical about zero. Fig.1 shows its upper part (curve 6). The set M is chosen /9/ by analyzing the motion of the plane after passing the end of the runway; if at the instant ϑ the lateral deviation $x_1(\vartheta)$ and

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its rate of change $x_2(\vartheta)$ are such that $(x_1(\vartheta), x_2(\vartheta)) \in M$, then after the instant ϑ rapid completion of the landing is guaranteed. If $(x_1(\vartheta), x_2(\vartheta)) \notin M$, then there is no such guarantee.

We shall understand by the admissible strategies of the first player /1/, the functions

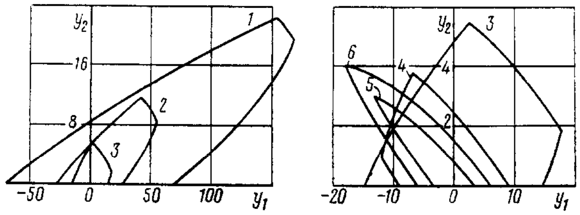

Fig.1

U placing every position $(t, x) \in [0, \vartheta] \times R^7$ in 1:1 correspondence with the number $U(t, x)$ satisfting the condition $|U(t, x)| \leqslant \mu(t)$. We denote by $x(\cdot, t_0, x_0, U, \Delta, v(\cdot))$ the motion of system (1.1) emerging at the instant $t_0 \in [0, \vartheta]$ from the point x_0 when the first player applies some admissible strategy U to a discrete control scheme /1/ with step Δ, and the second player realizes some measurable function of time $v(\cdot)$ satisfying the condition $|v(t)| \leqslant \nu$. Let

$$\gamma^{(1)}(t_0, x_0, U, \Delta) = \sup_{v(\cdot)} \varphi(x_1(\vartheta, t_0, x_0, U, \Delta, v(\cdot)), x_2(\vartheta, t_0, x_0, U, \Delta, v(\cdot))) \tag{1.5}$$

$$\Gamma^{(1)}(t_0, x_0) = \min_U \overline{\lim_{\Delta \to 0}} \gamma^{(1)}(t_0, x_0, U, \Delta)$$

The quantity $\Gamma^{(1)}(t_0, x_0)$ represents the optimal guarantee of the first player for the initial position (t_0, x_0).

Similarly, in case of the second player we shall represent the admissible strategies by the arbitrary functions V putting every position (t, x) in 1:1 correspondence with the number $V(t, x)$ satisfying the condition $|V(t, x)| \leqslant \nu$. We write

$$\Gamma^{(2)}(t_0, x_0) = \max_V \underline{\lim_{\Delta \to 0}} \gamma^{(2)}(t_0, x_0, V, \Delta) \tag{1.6}$$

where $\gamma^{(2)}$ is introduced, just as $\gamma^{(1)}$, with obvious changes in the definition.

We know /1/ that $\Gamma^{(1)}(t_0, x_0) = \Gamma^{(2)}(t_0, x_0)$ for a game of the type (1.1)–(1.4) for any initial position (t_0, x_0). Below, we shall write Γ instead of $\Gamma^{(1)}, \Gamma^{(2)}$. The function Γ is called the cost function of the game, and the strategies U°, V°, on which the minimum in (1.5) and maximum in (1.6) are attained, are the optimal strategies of the first and second player. In the general case the strategies U°, V° can depend on the initial position. From /4, 5/ it follows that a universal optimal strategy /1, 2/ of the first player exists in the game (1.1)–(1.4), i.e. strategy optimal for all initial positions. We shall describe an algorithm for the numerical construction of such a strategy. The universal strategy of the second player is constructed in exactly the same manner as that of the first player. It will not however be optimal in the strict sense, like the strategy of the first player. We shall consider a combined strategy of the first player representing a combination of the universal optimal strategy and a strategy base on a linear function.

When $k \to \infty$, system (1.1) transform into system (4.1) of /3/. The restrictions (1.2), (1.3) are identical with restrictions (4.2), and the payoff function (1.4) is close to payoff function (4.3). In /3/ an algorithm is given for the numerical construction of the universal strategy of the first player based on the method of extremal aiming /10/. However, since the problem (4.1)–(4.3) is not regular /1, 10/, the method does not ensure the optimal result for the first player. Neither is the differential game (1.1)–(1.4) studied below, regular.

2. Since the differential game (1.1)–(1.4) is a linear game with a fixed termination time ϑ, it has the following special feature: the payoff function φ depends on the values of only two components of the phase vector $x(\vartheta)$, namely on $x_1(\vartheta)$ and $x_2(\vartheta)$. This allows us /1, 11/ to pass, using the substitution $y(t) = X_{1,2}(\vartheta, t)\, x(t)$, from the game (1.1)–(1.4) to the equivalent second-order game

$$\begin{gathered} y^{\cdot} = D(t) u + E(t) v, \quad y \in R^2 \\ D(t) = \mu(t) X_{1,2}(\vartheta, t) B, \quad E(t) = \nu X_{1,2}(\vartheta, t) C \\ |u| \leqslant 1, \quad |v| \leqslant 1 \end{gathered} \tag{2.1}$$

Here $X_{1,2}(\vartheta, t)$ is a matrix consisting of the first two rows of the fundamental Cauchy matrix $X(\vartheta, t) = \exp A(\vartheta - t)$. The payoff function remains unchanged for the game (2.1). We denote the cost function by $\mathbf{\Gamma}$. The relation $\Gamma(t, x) = \mathbf{\Gamma}(t, X_{1,2}(\vartheta, t)\, x)$ holds.

The set of the level $W_c = \{(t, y) \in [0, \vartheta] \times R^2\colon \mathbf{\Gamma}(t, y) \leqslant c\}$, $c \geqslant 0$ of the cost function in game (2.1) is identical with the set of positional absorption of the specific set $M_c = cM$ at the instant ϑ, or, which is the same in the present case, with the maximum u-stable bridge arriving at M_c /1/ at the instant ϑ. The cross-section $W_c(t) = \{y \in R^2\colon (t, y) \in W_c\}$ is identical with the antisymmetrized integral /12/ of the set M_c for the game (2.1) in the interval $[t, \vartheta]$.

The cross-sections $W_c(t)$ were found using the standard program of constructing the positional absorption sets developed at the Institute of Mathematics and Mechanics, Ukrainian Scientific Centre, Academy of Sciences of the USSR. The cross-sections $W_c(t)$ are symmetrical about zero. Fig.1 shows the upper parts of the cross-sections obtained on a computer for $k = 1$, $c = 1$ for the times $t = 0, 5, 9, 11, 14, 15$ and labelled with the numbers 1–6 respectively. We note that $W_c(15) = M_c$.

The set W_c is the maximum "tube" in the space of variables t, y_1, y_2, from which the first player guarantees the arrival of system (2.1) to the set M_c irrespective of the actions of the second player. In terms of system (1.1) this means the following. If the initial position (t_0, x_0) is such that $X_{1,2}(\vartheta, t_0)x_0 \in W_c(t_0)$, then the first player has a strategy /1/ guaranteeing the inclusion $(x_1(\vartheta), x_2(\vartheta)) \in M_c$. If $X_{1,2}(\vartheta, t_0)x_0 \notin W_c(t_0)$, then there is no such strategy. Moreover, in this case a strategy of the second player exists /1/ which makes it impossible for the point $(x_1(\vartheta), x_2(\vartheta))$ to arrive at M_c no matter what the action of the first player. Thus, writing $c = 1$ we obtain with help of the set $W_1(0)$ a complete description of the totality of all initial states x_0 at the instant $t_0 = 0$, from which a rapid completion of the landing stage is guaranteed.

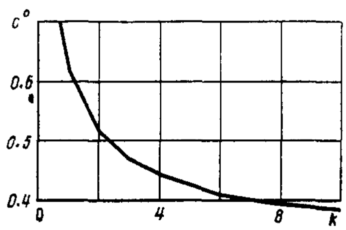

Fig.2

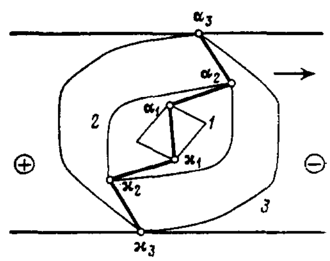

Fig.3

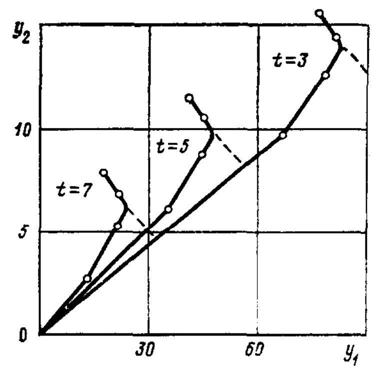

Fig.4

Let us denote by c° the smallest $c \geqslant 0$ for which $W_c(0) \neq \varnothing$. The quantity c° characterizes the potential possibilities of the control u during the time interval $[0, \vartheta]$. Fig.2 shows the numerically computed dependence of c° on the parameter k.

3. The differential game (1.1)–(1.4) represents a linear game with a fixed instant of termination and convex payoff function. The control parameter of the first (minimizing) player is a scalar. Under these conditions a stable universal optimal strategy of the first player exists /4, 5/. The strategy can be determined with help of a special surface $S^{(1)}$ in the space t, y_1, y_2, playing the part of the switchover surface for the control of the first player.

Let us describe schematically the method of constructing the cross-sections of the surface $S^{(1)}$. Suppose we wish to construct the cross-section $S^{(1)}(t)$ for the time instant $t \in [0, \vartheta]$ (i.e. the switchover line for the instant t). We denote by c_t the smallest $c \geqslant 0$, for which $W_c(t) \neq \varnothing$. Choosing a value $c_* \geqslant c_t$ sufficiently close to c_t and a sufficently large value $c^* > c_*$, we define on $[c_*, c^*]$ an ordered set of increasing numbers $c1 = c_*, c2, \ldots, cn = c^*$. We construct for every ci a set $W_{ci}(t)$. The set $W_{ci}(t)$ is closed, convex and bounded. Traversing its boundary in the clockwise direction we find the point $\varkappa_i$ (or respectively α_i) at which the scalar product of the vector $D(t)$ and the vector of the external normal to the boundary changes its sign from plus to minus (from minus to plus). Going through i from 1 to n, we obtain the sets $\varkappa_1, \varkappa_2, \ldots, \varkappa_n$ and $\alpha_1, \alpha_2, \ldots, \alpha_n$. Connecting, one after the other, the points of these sets with the segments and $\varkappa_1$ with α_1, we obtain the polygonal line, and we use it as the line $S^{(1)}(t)$. The degree of closeness to the "perfect" switchover line is higher, the smaller the diameter of partitioning the segment $[c_*, c^*]$ by the numbers ci and the closer c_* to c_t. Fig.3 illustrates qualitatively the construction of the switchover line for $n = 3$. The sets $W_{c1}(t)$, $W_{c2}(t)$ and $W_{c3}(t)$ are denoted by the numbers 1, 2, 3, and the direction of the vectors $D(t)$ is indicated by an arrow.

Let $\Lambda^{(1)}(t)$ be a strip in the y_1, y_2 plane, composed of straight lines passing through the set $W_{c^*}(t)$ in a direction parallel to the vector $D(t)$. The line $S^{(1)}(t)$ divides the strip $\Lambda^{(1)}(t)$ into two parts. We denote the part into which the vector $D(t)$ is pointing by $S_-^{(1)}(t)$, and the other by $S_+^{(1)}(t)$. (In Fig.3 these parts are marked by the minus and plus sign respectively). If the point $y(t) = X_{1,2}(\vartheta, t)x(t)$ arrives at the instant t at $S_-^{(1)}(t)$, then the optimal control $U^\circ(t, x(t))$ of the first player in system (1.1) must be taken equal to $\mu(t)$. If the point arrives at $S_+^{(1)}(t)$, then the control is $\mu(t)$. At the line itself any control from the segment $[-\mu(t), \mu(t)]$ will suffice. We stress that the size of the strip $\Lambda^{(1)}(t)$ depends on c^*. This value must be chosen (the same for all t, or depending on t) so that any motion $y(t) = X_{1,2}(\vartheta, t)\, x(t)$ with any initial condition x_0 at the instant t_0, chosen form a reasonable bounded domain of possible initial positions, would find itself at the instant t within the strip $\Lambda^{(1)}(t)$.

Having found the lines $S^{(1)}(t)$ for all instances $t \in [0, \vartheta]$, we have in fact specified the universal optimal strategy U°. Since the strategy is realized in the discrete control scheme, it follows in fact that it is necessary to construct the lines $S^{(1)}(t)$ only for those instances at which the control will be chosen for the first player.

Fig.4 shows the switchover lines of the first player computed numerically at $k = 1$ for the time instances $t = 3, 5, 7$ (the solid lines). Since the cross-sections $W_c(t)$ are symmetrical about zero, it follows that so are the switchover lines. Therefore only the upper parts of the lines are shown. In computing the lines we used 17 values of the parameter c, ranging from $c_* = 0{,}63$ to $c^* = 1$. The points marked along the lines correspond to $c = 0{,}63; 0{,}66; 0{,}79; 1$ and are ordered. The last point on each line corresponds to $c = 1$. The qualitative character of the switchover lines of the first player is the same for all $t \in [0, \vartheta]$. Namely, the upper part of the line $S^{(1)}(t)$ is divided, for all t, by the characteristic corner point, into two segments, each segment being nearly linear. The control $U^\circ(t, x)$ takes the value $-\mu(t)$, when the point $X_{1,2}(\vartheta, t)\, x$ lies to the right of the switchover line, and $\mu(t)$ when it lies to the left of it.

4. The definition of the sets W_c, $c \geqslant 0$, and the properties of universality of the strategy U° discussed in the previous section imply, that, if having fixed the number c' we choose the control of the first player at the given instant t with help of the strategy U° only when $y(t) = X_{1,2}(\vartheta, t)\, x(t) \notin W_{c'}(t)$, and specify the control in a different manner when $y(t) \in W_{c'}(t)$, then the result guaranteed to the first player by this control will be identical with the optimal (i.e. the cost of the game) in the case when $y(t_0) = X_{1,2}(\vartheta, t_0)\, x_0 \notin W_{c'}(t_0)$, and will not exceed c' when $y(t_0) \in W_{c'}(t_0)$. We shall use this property to introduce a combined strategy for the first player, putting $c' = c^\circ$.

Let us consider the linear function

$$f(x) = (-0{,}1x_1 - 1{,}5x_2 + 5x_4)/57{,}3$$

and the function

$$F(t, x) = \begin{cases} \mu(t), & f(x) > \mu(t) \\ f(x), & |f(x)| \leqslant \mu(t) \\ -\mu(t), & f(x) < -\mu(t) \end{cases} \tag{4.1}$$

The control laws resembling F (i.e. linear laws with restricted amount of bank) are used in the flight systems of civil aircraft to stabilize the lateral deviations /7, 13/.

Let us denote by $G(t)$ a circle of maximum radius inscribed into $W_{c^\circ}(t)$. We introduce the universal combined strategy U' as follows: if the position (t, x) is such that $X_{1,2}(\vartheta, t)\, x \notin G(t)$, we put $U'(t, x) = U^\circ(t, x)$; if $X_{1,2}(\vartheta, t)\, x \in G(t)$, then we write $U'(t, x) = F(t, x)$.

The strategy U' guarantees the first player, at any initial state x_0, at the instant $t_0 = 0$ a result equal to the optimal result. Thus for the initial positions (t_0, x_0) at $t_0 = 0$ the strategy will not be "worse" than the strategy U°. The strategy U' can be found to be preferable to the strategy U°, since its realization does not result in frequent switchover of the control of the first player (slippage mode) when the deviations $x(t)$ from zero in the process of motion, are small.

5. The control parameter v of the second player in system (1.1), as well as the control parameter u of the first player, are scalar quantities. Just as we constructed the switchover surface $S^{(1)}$ in Sect.3 for the first player, we can construct in the space t, y_1, y_2 a switchover surface $S^{(2)}$ for the second player, constructing it from the lines $S^{(2)}(t)$. We only need to replace the vector $D(t)$ from system (2.1) by the vector $E(t)$.

Let us introduce the strategy V^* of the second player based on the switchover surface. Let t be any instant from $[0, \vartheta]$. We define the strip $\Lambda^{(2)}(t)$ in the y_1, y_2 plane, just as we defined the strip $\Lambda^{(1)}(t)$, replacing $D(t)$ by $E(t)$. The line $S^{(2)}(t)$ divides $\Lambda^{(2)}(t)$ into two parts. We denote by $S_+^{(2)}(t)$ the part into which the vector $E(t)$ is directed, and the other part by $-S_-^{(2)}(t)$. If x is such that $X_{1,2}(\vartheta, t)\, x \in S_+^{(2)}(t)$, we write $V^*(t, x) = \nu$, and if $X_{1,2}(\vartheta, t)\, x \in S_-^{(2)}(t)$, then we write $V^*(t, x) = -\nu$. On the line $S^{(2)}(t)$ itself the value of $V^*(t, x)$ is arbitrarily chosen from the interval $[-\nu, \nu]$.

The strategy V^* is determined in exactly the same manner as U°, but unlike the latter it is not strictly optimal.

The reason for this is the following. When we choose arbitrarily the control of the second player at the switchover surface, we do not exclude, under ideal conditions (i.e. without allowing for the error of approximation), the motion $(t, y(t)) = (t, X_{1,2}(\vartheta, t)\, x(t))$, along $S^{(2)}$. The cost of the game along such motions may diminish. In other words, the second player may lose by allowing the motions along the surface $S^{(2)}$. This distinguishes it from the case of motion along the surface $S^{(1)}$, when the point of view of the first player is considered /4, 5/. Introducing a measure of definiteness into specifying the control of the second player on the surface $S^{(2)}$ will not change anything: in forming this control with the help of the switchover surface constructed in an approximate manner, the second player will err compared with the choice made relative to the perfect surface, and this may lead to the appearance of the slippage mode. However, for this to happen, the first player must behave very "skilfully". If we neglect the possibility of the slippage mode appearing on the surface $S^{(2)}$ in which the cost of the game along the motion could diminish, then we can regard the strategy V^* as practically optimal.

The qualitative character of the switchover lines of the second player is more complex than that of the switchover lines of the first player. Computer simulations show that the interval $[0, \vartheta]$ is partitioned into several characteristic subintervals. On one of them the lines $S^{(2)}(t)$ coincide completely with the lines $S^{(1)}(t)$, and on the other segments the lines $S^{(2)}(t)$, $S^{(1)}(t)$ overlap partially. If we proceed from the origin of coordinates, we see that their upper parts coincide up to the corner point of the switchover lines of the first player, and then diverge in opposite directions. In particular, the instances $t = 3, 5, 7$. at $k = 1$ refer to segments of this type. The corresponding upper parts of the lines $S^{(2)}(t)$ are shown in Fig.4. The dashed lines denote the segments not belonging to $S^{(1)}(t)$. In addition to those described, we have short intervals where the lines of the second player change rapidly, from coincidence with the lines of the first player, to positions resembling those in Fig.4 and vice-versa.

6. Let us give the results of numerical modelling of the motions of system (1.1) for two initial states x_0 at the instant $t_0 = 0$. The first (point a) is characterized by the lateral deviation $x_{01} = 50$ m, and the remaining coordinates $x_{02}, \ldots, x_{07}$ are zero

	$x_0 = a$		$x_0 = b$	
	U'	F	U'	F
V^*	0.75	2.20	0.63	2.88
$v \equiv 0$	0.02	1.75	0	0
$v \equiv 10$	0.65	1.66	0.66	0.10
v^*	0.23	1.32	0.24	0.38

The second initial state is the point b with zero coordinates. The coefficient k on the right hand side of (1.1) was taken as unity.

The cost of the game $\Gamma(0, a)$ for the initial position $(0, a)$ is equal to 0.69. The value was found experimentally by choosing the value c for which the point $X_{1,2}(\vartheta, 0)\, a$ lies at the boundary of the set $W_c(0)$. For the initial position $(0, b)$ the cost $\Gamma(0, b)$ is identical with $c^\circ = 0.62$.

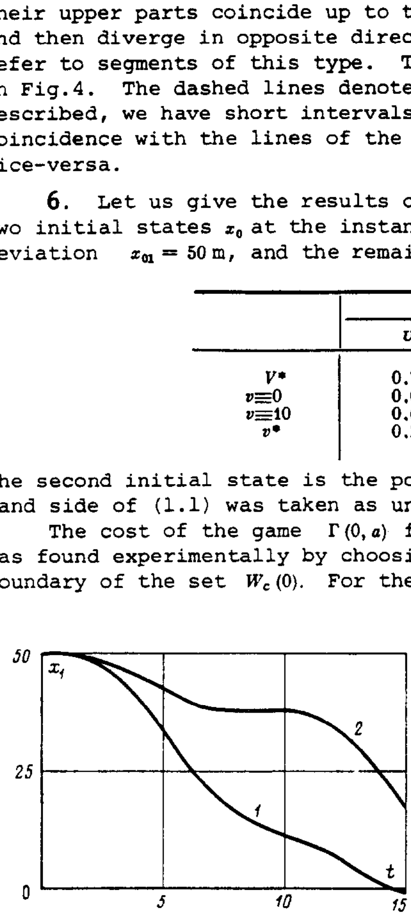

Fig.5

The table gives the values of the payoff function $\varphi(x_1(\vartheta), x_2(\vartheta))$ obtained at the instant ϑ for various types of player control. The expression $v \equiv 0$ ($v \equiv 10$) denotes the constant control of the second player equal to zero (10 m/sec respectively). The symbol v^* corresponds to the control of the second player random with respect to time. The function v^* was piecewise constant in t, with a step of 1 sec, and the value of the function at every constant interval was chosen using a random number generator realizing the uniform distribution over the segment $[-10, 10]$. The symbol F denotes the strategy of the first player, given by the formula (4.1) and used for all positions (t, x).

To realize the strategies U', V^*, we used the switchover lines computed for 9 values of the parameter c: 0.64; 0.70; 0.76; 0.82; 0.88; 0.94; 1; 3; 5. The step Δ of discrete control schemes was equal to 0.05. The same step was used when realizing the strategy F. Increasing the step of the discrete scheme of the first player to 0.1, gave no appreciable change in the payoff.

Fig.5 shows graphs of the change in lateral deviation $x_1(t)$ with time t for the initial position $(0, a)$, when the strategy V^* is used for the second player. Curve *1* corresponds to the strategy U' of the first player, and curve *2* to the strategy F.

The results obtained show that under the extremal perturbation from V^* (the first line in the table) the linear control law F does not guarantee a successful landing approach. The combined method U' on the other hand, does give such a guarantee. Linear control does not allow successful completion of the process even when there are no extremal perturbations. Combined control on the other hand, will deal successfully with the case of large initial deviations (the second column).

In conclusion, we turn our attention once again to the assumption that the termination time is fixed in formulating the modelling problem in question. A rigorous solution of the analogous problem where the instant of termination (treated as the instant when the runway has been traversed) is not fixed a priori, but may lie within certain limits, is essentially more difficult, since the possibility of passing to a second-order game, which would be equivalent to the initial game, is lost.

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